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On Finding Integer Solutions to Non-homogeneous Ternary Bi-quadratic Equation $3(x^2+y^2)-2xy=11z^4$

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Abstract: **This paper concerns with the problem of obtaining non-zero distinct integer solutions to the non-homogeneous ternary bi-quadratic equation $3(x^2+y^2)-2xy=11z^4$. Different sets of integer solutions are illustrated.**

Keywords: **non-homogeneous bi-quadratic, ternary bi-quadratic integer solutions.**

I. INTRODUCTION

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-28] for a few problems on Biquadratic equation with 3 unknowns. This paper concerns with yet another interesting Biquadratic Diophantine equation with three variables given by $3(x^2+y^2)-2xy=11z^4$ for determining its infinitely many non-zero distinct integral solutions

II. METHOD OF ANALYSIS

The non-homogeneous ternary bi-quadratic equation under consideration is

$$3(x^2+y^2)-2xy=11z^4 \tag{1}$$

Introduction of the linear transformations

$$x=2(u+v),\, y=2(u-v),\, z=2w,\, u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2+2v^2=11w^4 \tag{3}$$

Solving (3) for u, v, w through different ways as illustrated below and using (2), one

Obtains the corresponding integer solutions to (1).

Way 1:

Let

$$w = a^2 + 2b^2 \tag{4}$$

Write 11 on the R.H.S. of (3) as

$$11 = (3 + i\sqrt{2})(3 - i\sqrt{2}) \tag{5}$$

Substituting (4) & (5) in (3) and employing the method of factorization, consider

$$u + i\sqrt{2}v = (3 + i\sqrt{2})(a + i\sqrt{2}b)^4 \tag{6}$$

On equating the real and imaginary parts in (6), the values of u, v are obtained.

In view of (2), the corresponding integer solutions to (1) are obtained as

$$x = 8(a^4 - 12a^2b^2 + 4b^4) + 2(4a^3b - 8ab^3),$$
$$y = 4(a^4 - 12a^2b^2 + 4b^4) - 10(4a^3b - 8ab^3),$$
$$z = 2(a^4 + 2b^2)$$

Note 1:

The integer 11 on the R.H.S. of (3) is also represented as

$$11 = \frac{(7 + i5\sqrt{2})(7 - i5\sqrt{2})}{9}$$

Repetition of the above process leads to a different set of integer solutions to (1).

Way 2:

Rewrite (3) as

$$u^2 + 2v^2 = 11w^4 * 1 \tag{7}$$

Consider 1 on the R.H.S. of (7) as

$$1 = \frac{(1 + i2\sqrt{2})(1 - i2\sqrt{2})}{9} \tag{8}$$

Following the procedure as in Way1, the corresponding integer solutions to (1) are found to be

$$x = 54(6(a^4 - 12a^2b^2 + 4b^4) - 15(4a^3b - 8ab^3)),$$
$$y = 54(-8(a^4 - 12a^2b^2 + 4b^4) - 13(4a^3b - 8ab^3)),$$
$$z = 18(a^2 + 2b^2)$$

Note 2:

The integer 1 on the R.H.S. of (8) is also expressed as

$$1 = \frac{(2r^2 - s^2 + i2\sqrt{2}rs)(2r^2 - s^2 - i2\sqrt{2}rs)}{(2r^2 + s^2)^2}$$

Repeating the above process,a different set of solutions to (1) are obtained.

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Way 3:

Express (3) in the form of ratios as

$$\frac{u+3w^2}{w^2+v}=\frac{2(w^2-v)}{u-3w^2}=\frac{\alpha}{\beta},\beta\neq 0 \tag{9}$$

Solving the above system of double equations (9),one has

$$u=3\alpha^2+4\alpha\beta-6\beta^2,\ v=-\alpha^2+6\alpha\beta+2\beta^2,\ w=s^2+2r^2 \tag{10}$$

where

$$\alpha=2r^2-s^2,\beta=2rs$$

From (10) and (2), the corresponding integer solutions to (1) are given by

$$x=2(8r^4+2s^4-24r^2s^2+40r^3s-20rs^3),$$
$$y=2(16r^4+4s^4-48r^2s^2-8r^3s+4rs^3),$$
$$z=2(2r^2+s^2)$$

Note 3:

One may also write (3) in the form of ratios as

$$\frac{u+3w^2}{2(w^2+v)}=\frac{(w^2-v)}{u-3w^2}=\frac{\alpha}{\beta},\beta\neq 0,$$
$$\frac{u+3w^2}{(w^2-v)}=\frac{2(w^2+v)}{u-3w^2}=\frac{\alpha}{\beta},\beta\neq 0,$$
$$\frac{u+3w^2}{2(w^2-v)}=\frac{(w^2+v)}{u-3w^2}=\frac{\alpha}{\beta},\beta\neq 0,$$

The repetition of the above process gives three more integer solutions to (1).

Way 4:

The substitution of the transformations

$$w^2=X+2T,\ v=X+11T,\ u=3U \tag{11}$$

in (3) leads to

$$X^2=U^2+22T^2 \tag{12}$$

which is satisfied by

$$T=2rs,\ U=22r^2-s^2,\ X=22r^2+s^2 \tag{13}$$

Substituting (13) in (11) ,note that

$$u=3(22r^2-s^2),\ v=(22r^2+s^2+22rs) \tag{14}$$

and

$$w^2=(22r^2+s^2+4rs) \tag{15}$$

Treating (15) as a quadratic in s and solving for s, we have

$$w=(18p^2+q^2),r=2pq,s=-4pq\pm(18p^2-q^2)$$

Using the above values of r,s in (14) and in view of (2), the corresponding two sets of integer solutions to (1) are as presented below:

Set 1:

$$x=2(-648p^4-2q^4+216p^2q^2+1080p^3q-60pq^3,$$
$$y=2(-1296p^4-4q^4+432p^2q^2-216p^3q+12pq^3,$$
$$z=2(18p^2+q^2)$$

Set 2:

$$x=2(-648p^4-2q^4-136p^2q^2-1080p^3q+60pq^3,$$
$$y=2(-1296p^4-4q^4+256p^2q^2+216p^3q-12pq^3,$$
$$z=2(18p^2+q^2)$$

Way 5:

Represent (12) as the system of double equations as below in Table:1

Table:1-system of double equations

System	I	II	III	IV
$X+U$	T^2	$11T^2$	$22T$	$11T$
$X-U$	22	2	T	$2T$

Solving each of the above system of equations, the corresponding values of X,T,U are obtained.

From (11),the values of u,v,w are found .In view of (2), the corresponding integer solutions to (1) are determined. For brevity ,the integer solutions obtained from each of the above four systems are exhibited as follows:

Solutions from system I:

$$x_{n+1}=2(8k^2{}_{n+1}+22k_{n+1}-22),$$
$$y_{n+1}=2(4k^2{}_{n+1}-22k_{n+1}-44)\ ,$$
$$z_{n+1}=9f_n+6\sqrt{2}g_n$$

where

$$k_{n+1}=3f_n+\frac{9}{2\sqrt{2}}g_n-1,$$
$$f_n=(3+2\sqrt{2})^{n+1}+(3-2\sqrt{2})^{n+1},$$
$$g_n=(3+2\sqrt{2})^{n+1}-(3-2\sqrt{2})^{n+1}$$

Solutions from system II:

$$x_{n+1} = 2(88k^2_{n+1} + 22k_{n+1} - 2),$$
$$y_{n+1} = 2(44k^2_{n+1} - 22k_{n+1} - 4),$$
$$z_{n+1} = \frac{1}{2} f_n + \frac{1}{\sqrt{22}} g_n$$

where

$$k_{n+1} = \frac{1}{22}(f_n + \frac{11}{\sqrt{22}} g_n - 2,$$
$$f_n = (197 + 42\sqrt{22})^{n+1} + (197 - 42\sqrt{22})^{n+1},$$
$$g_n = (197 + 42\sqrt{22})^{n+1} - (197 - 42\sqrt{22})^{n+1}$$

Solutions from system III:

$$x = 216*27k^2, y = 36*27k^2, z = 54k$$

Solutions from system IV:

$$x = 124*17k^2, y = -16*17k^2, z = 68k$$

III. CONCLUSION

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous bi-quadratic diophantine equation with three unknowns given by $xz(x-z) = y^4$.One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multi-variables

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